Geographic and Process Information for Chemical Plant Layout Problems

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Geographic information systems (GIS) introduced here offer a source of information for problems with a geographic dimension, such as layout and environmental decision making. A mixed-integer linear-programming formulation of a chemical-plant layout problem was developed, and is presented along with an existing formulation. The effectiveness of these formulations is highly problem- and data-specific, and a combination of the two formulations can be derived that is more effective on certain problems than either formulation alone. The formulations are ineffective for some problems due to the highly redundant search space engendered by symmetries in the layout—symmetries that can be eliminated by a more careful analysis of the situation using location-specific information such as existing units and facilities, geographic constraints, wind, elevation, and soil conditions. Different classes of such information can be captured from a GIS and represented within the mixed-integer linear-programming formulation. Two cases studies presented illustrate the formulations and the use of geographic information for chemical-plant layout.

Introduction

Process-plant layout is an important issue in chemical-plant design, and diverse sources of knowledge are required to end up with a sound layout. This is because the layout decisions are driven by several factors: spatial limitations, efficiency, cost, safety, construction, maintenance and emergency considerations, environmental concerns, operational requirements, or space needs for future expansion (Mecklenburgh, 1985). Most of these considerations require information that depends on the location of the equipment and facilities. The interactions between existing and planned equipment and/or facilities, safety and maintenance information of the units, or information needed for off-site consequence analysis (EPA, 1996) should be systematically collected, stored, and retrieved during present and future decision making. A system that handles geographic information and is integrated with decision support tools will be helpful to the chemical engineers for a wide variety of problems. This article describes an approach that combines geographic information with an accurate decision-making procedure for the facility layout problem. This approach enables us to generate new location-based information and to maintain records for future use. The geographic information system (GIS) is used as the central data structure to support facility-layout decision making and mathematical programming as the decision-making procedure.

The facility layout problem can be abstractly defined as locating a given number of facilities in the plane to minimize an objective value that depends on a distance measure between units, subject to a variety of constraints on distances. Each facility has a given area and shape, and the cost of interactions between every facility pair and the minimum distance between facilities are given. Frequently the problem is stated as an optimal assignment of the facilities to *a priori* known locations, which is formulated as a quadratic assignment problem (QAP) (Koopmans and Beckmann, 1957). This formulation cannot represent situations where the possible facility locations are not known *a priori*, and reformulations of the problem to address this have been proposed (Kusiak and Heragu, 1987; Welgama and Gibson, 1995). A mixed-

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integer linear model version of the QAP was formulated such that locations are given as points in a two-dimensional plane, and Manhattan distances were used to describe the separation between points (Love and Wong, 1976). In other mixed-integer linear models Manhattan distances are used and models usually require *a priori* known locations using a grid-based approach. Drezner (1980) reformulated the problem to use circular units and Euclidean distance measure. Although initial specification of locations is not required, this formulation resulted in an NLP, which is nonconvex, since the constraints defining the set of feasible points is reverse convex (Horst et al., 1995), although the objective function is convex for nonnegative cost coefficients. Solved by the lagrangean differential method (Drezner, 1980), Drezner's NLP formulation gave near-optimal results for fairly large problems in reasonable time. The requirement to know the potential locations *a priori* in mixed-integer linear models was eliminated by Heragu and Kusiak (Heragu and Kusiak, 1991). They proposed formulations to lay out the facilities on a (continuous) plane and applied a penalty method to an unconstrained version of the formulation to derive suboptimal solutions quickly.

In the chemical engineering literature the formulations just summarized, their variations, and reformulations, are used in several layout-related problems. One of these is a grid-based formulation similar to Love and Wang's linearization of QAP that is used for multipurpose batch plants and extended to three dimensional space (Georgiadis et al., 1997). More recently a continuous-domain model is described and a small number of 8–12-unit problems are efficiently solved (Papageorgiou and Rotstein, 1998). Another example from the chemical engineering literature is extending an NLP-based formulation to deal with safety aspects of chemical processes (Penteado and Ciric, 1996). In addition to these applications, layout formulations are also a component of pipeless-plant design and scheduling (Realff et al., 1996, 1998).

However, the use of the facility layout formulations, in coordination with relevant information systems has not yet attracted academic attention. Two related approaches, namely the utilization of the solutions from optimization in computer-aided drafting procedures (Gunn and Al-Asadi, 1987) and the advantages of using a graphical interface to monitor the progress of a stochastic algorithm (Castell et al., 1998) do not aim to coordinate an information system with decision making. We claim that, since most of the layout problems are dependent on their actual physical location, GIS can be employed to capture information and enhance problem solving.

GIS is a computer-based database management system for capture, storage, retrieval, analysis, and display of locationally defined (spatial) data with the unique visualization and geographic analysis benefits offered by maps (Huxhold, 1991). Spatial problem solving, such as choosing a location to meet specified requirements, is also one of the many features of geographic information systems (Laurini and Thompson, 1992). The single most distinguishing characteristic of a geographic information system is its ability to integrate information from many different sources and at many different levels of detail. This characteristic enables GIS to have a wide range of applications from finding solid waste collection routes to facility management (*About GIS*, 1998). It has been used to integrate elevation, slope, land use, and soil-type data with meteorological and hydrological response submodels. These linked environmental process models can then be used to simulate specific conditions (such as flood) and policies (such as land use policy) (Nyerges, 1993). GIS is also used in optimization applications that have a critical geographic dimension (Camm et al., 1997; Özyurt and Realff, 1998). This article extends GIS-optimization integration and presents a procedure for using geographic information for the layout problem. In this application, the information system can supply the decision procedure with existing unit interactions and information about facilities outside the plant boundary. In-plant information can be collected and linked with a relevant map of the facility, which will also be used as a visual aid to find heuristic rules to decrease the problem size or character. The information about the surroundings can help to locate existing facilities that can generate constraints on in-plant locations (such as a unit with high noise levels cannot be close to the administration building of the nearby facility), or about the parameters to calculate hazardous distances such as wind speed and temperature. Moreover, the information system can be used to integrate different levels of process-plant layout decisions, such as site selection, site layout, and plot layout (Mecklenburgh, 1985).

In the following section we define the overall problem, followed by the mathematical formulation of the layout problem. After solving some examples and discussing the results in the third section, we introduce GIS to integrate the spatial information with chemical-plant layout formulation in the fourth section. We give two case studies in the fifth section, and finally conclude with a discussion and future directions.

Problem Statement and Layout Formulations

Formal statement of the problem is as follows:

Given:

- Representation of the area of interest such as a map, CAD file, or an aerial image.
- Attribute data about the existing facilities, processes, and environment.
- Attribute data about new facilities, units such as the external dimensions of the units (assuming rectangular borders); connection (interaction) matrix between units; and cost of piping (connection cost).

Determine: the x and y coordinates of each unit, such that the total cost of interactions between units is minimized, environmental and safety constraints are satisfied, and new maps for future use are established.

There are two basic types of layout formulation, grid based and continuous plane. In grid-based formulation, either the grid locations are larger than the units, leading to a coarse grid, or the units cover multiple grid locations, leading to a more complex formulation. The first case results in suboptimal solutions, while the second case requires excessive computer time and still does not allow units to be located anywhere in the available space (Figure 1). These deficiencies of the grid-based formulations to describe the real nature of the process layout formulation have been highlighted in various studies (Heragu and Kusiak, 1991; Papageorgiou and Rotstein, 1998). Thus, in this article, the continuous-plane for-

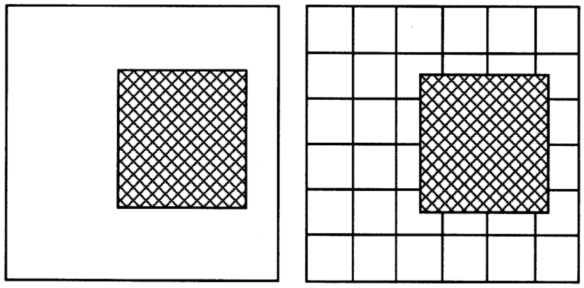


Figure 1. Any location possible in continuous plane (a) except for grid based formulations (b).

mulation is adopted. We have another reason to use continuous-plane formulations: physical locations retrieved from a geographic information system are usually given as points with real x and y coordinates or latitude and longitude information, which must be coerced to the grid points for the other formulation to be applied.

The basic equations for a continuous-plane formulation can be defined using absolute-value functionals. The formulation consists of a set of constraints forbidding overlap between units and allowing only a possible distance between units and another set of constraints ensuring that the units are located within the boundaries of the floor plan. The parameters and variables of Figure 2 are listed in the Notation section.

A second decision in the layout formulations is the definition of the distance measure. The Manhattan distance measure allows us to describe the piping connections between units realistically and can be formulated within a mixed-integer linear programming (MILP) model. For these two reasons we will use Manhattan distance as our measure in the layout formulation.

The general formulation of the layout problem with Manhattan distance measure consists of two types of constraint sets: nonoverlapping and floorplan constraints. In addition, the units can be given a variable orientation, rotated 90 deg with respect to each other (Papageorgiou and Rotstein, 1998).

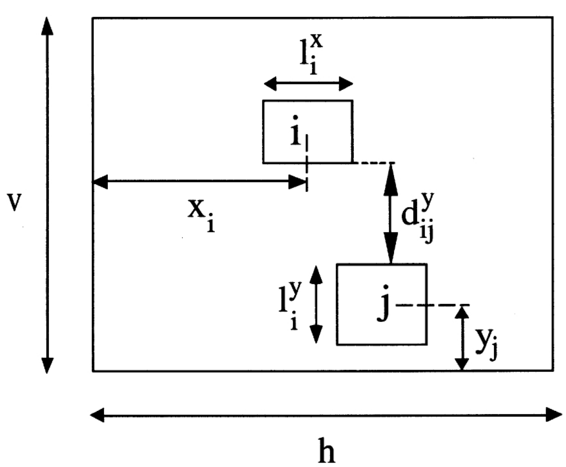


Figure 2. Variables.

Nonoverlapping constraints

Nonoverlapping constraints prevent any two units from occupying the same location and keep them separated by more than the given minimum allowable distance. It is sufficient for any unit pair to satisfy these constraints in only one direction (in two-dimensional case x- or y-direction). The alignment is allowed in the other direction:

$$|x_i - x_j| \geq \frac{1}{2}\left(l_i^x + l_j^x\right) + d_{ij}^x \quad \text{or} \quad |y_i - y_j| \geq \frac{1}{2}\left(l_i^y + l_j^y\right) + d_{ij}^y$$

$$\text{for} \quad i = 1 \cdots n-1, \qquad j = i+1 \cdots n.$$

Floor plan constraints

It is assumed that facilities have a prespecified border, which can be described (approximated) with rectangles. The simplest form of these constraints can be stated as follows:

$$\frac{1}{2}l_i^x \leq x_i \leq h - \frac{1}{2}l_i^x \qquad \text{for} \quad i = 1 \cdots n$$

$$\frac{1}{2}l_i^y \leq y_i \leq v - \frac{1}{2}l_i^y \qquad \text{for} \quad i = 1 \cdots n.$$

Objective

A reasonable objective will contain a distance and interaction-dependent cost function. Using these constraints and objective the original model becomes the following:

P1

$$\min \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} C_{ij}\left(|x_i - x_j| + |y_i - y_j|\right) \tag{1}$$

$$|x_i - x_j| \geq \frac{1}{2}\left(l_i^x + l_j^x\right) + d_{ij}^x$$

or

$$|y_i - y_j| \geq \frac{1}{2}\left(l_i^y + l_j^y\right) + d_{ij}^y$$

$$\text{for} \quad i = 1 \cdots n-1, \qquad j = i+1 \cdots n \tag{2}$$

$$\frac{1}{2}l_i^x \leq x_i \leq h - \frac{1}{2}l_i^x \quad \text{for} \quad i = 1 \cdots n \tag{3}$$

$$\frac{1}{2}l_i^y \leq y_i \leq v - \frac{1}{2}l_i^y \quad \text{for} \quad i = 1 \cdots n \tag{4}$$

$$x_i, y_i \geq 0 \qquad \text{for} \quad i = 1 \cdots n \tag{5}$$

However to solve **P1** with standard mathematical programming algorithms a transformation into an MILP is needed. This transformation should first linearize the absolute-value functionals and then describe the disjunction in constraint 2. A transformation that adopts a representation suitable for palletization in warehouses is given in Papageorgiou and Rotstein (1998), which will be referred as **FL** throughout this article. This formulation used "big M" to describe the disjunctive constraints, which frequently suffers from poor LP

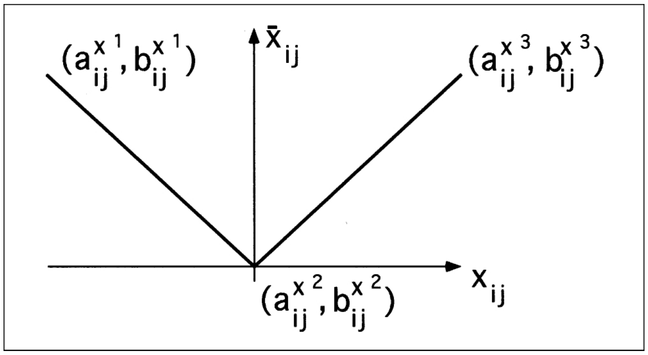


Figure 3. Absolute-value functionals.

relaxation (Raman and Grossmann, 1994). Another approach is to utilize the representation of the graph for a piecewise-linear function (Jeroslow, 1989) to linearize the absolute-value functionals, shown in Figure 3 and use a tighter formulation for the disjunction in constraints 2. This type of formulation might improve the lower bound in the branch-and-bound procedure; however, it results in more variables and constraints.

The formulation of the problem using disjunctive convex hull and the graph of the absolute-value function, without allowing equipment orientation to vary, uses the following additional parameters and variables.

Parameters

$a_{ij}^{x1},\ a_{ij}^{x2},\ a_{ij}^{x3},\ b_{ij}^{x1},\ b_{ij}^{x2},\ b_{ij}^{x3}$ = parameters required to linearize absolute-value functionals in the x-direction (see Figure 3). They are defined as

$$\left(a_{ij}^{x1}, a_{ij}^{x2}, a_{ij}^{x3}\right) = \left[-\left(h - \frac{l_i^x + l_j^x}{2}\right), 0, h - \frac{l_i^x + l_j^x}{2}\right]$$

$$\left(b_{ij}^{x1}, b_{ij}^{x2}, b_{ij}^{x3}\right) = \left[h - \frac{l_i^x + l_j^x}{2}, 0, h - \frac{l_i^x + l_j^x}{2}\right].$$

$a_{ij}^{y1}, a_{ij}^{y2}, a_{ij}^{y3}, b_{ij}^{y1}, b_{ij}^{y2}, b_{ij}^{y3}$ = parameters required to linearize absolute-value functionals in the y-direction. They are defined as

$$\left(a_{ij}^{y1}, a_{ij}^{y2}, a_{ij}^{y3}\right) = \left[-\left(v - \frac{l_i^y + l_j^y}{2}\right), 0, v - \frac{l_i^y + l_j^y}{2}\right]$$

$$\left(b_{ij}^{y1}, b_{ij}^{y2}, b_{ij}^{y3}\right) = \left[v - \frac{l_i^y + l_j^y}{2}, 0, v - \frac{l_i^y + l_j^y}{2}\right].$$

Variables

$\bar{x}_{ij}$ = distance between units i and j in the x-direction

$\bar{y}_{ij}$ = distance between units i and j in the y-direction

$$m_{ij}^x = \begin{cases} 1 & \text{if alignment disallowed in the } x\text{-direction} \\ 0 & \text{otherwise} \end{cases}$$

$$m_{ij}^y = \begin{cases} 1 & \text{if alignment disallowed in the } y\text{-direction} \\ 0 & \text{otherwise} \end{cases}$$

$\gamma_{ij}^{x1}, \gamma_{ij}^{x2}$ = variables to determine the linear combination of endpoints for linearization (x-direction)

$\gamma_{ij}^{y1}, \gamma_{ij}^{y2}$ = variables to determine the linear combination of endpoints for linearization (y-direction)

$$\lambda_{ij}^x = \begin{cases} 1 & \text{if unit } j \text{ to right of unit } i \\ 0 & \text{if unit } j \text{ to left of unit } i \end{cases}$$

$$\lambda_{ij}^y = \begin{cases} 1 & \text{if unit } j \text{ above unit } i \\ 0 & \text{if unit } j \text{ below unit } i \end{cases}$$

The formulation can be stated as follows:

FD1 $$\min \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} C_{ij}\left(\bar{x}_{ij} + \bar{y}_{ij}\right) \tag{6}$$

$$x_i - x_j = \gamma_{ij}^{x1} a_{ij}^{x1} + \left(\lambda_{ij}^x - \gamma_{ij}^{x1}\right) a_{ij}^{x2} + \gamma_{ij}^{x2} a_{ij}^{x2} + \left(1 - \lambda_{ij}^x - \gamma_{ij}^{x2}\right) a_{ij}^{x3} \tag{7}$$

$$\bar{x}_{ij} = \gamma_{ij}^{x1} b_{ij}^{x1} + \left(\lambda_{ij}^x - \gamma_{ij}^{x1}\right) b_{ij}^{x2} + \gamma_{ij}^{x2} b_{ij}^{x2} + \left(1 - \lambda_{ij}^x - \gamma_{ij}^{x2}\right) b_{ij}^{x3} \tag{8}$$

$$y_i - y_j = \gamma_{ij}^{y1} a_{ij}^{y1} + \left(\lambda_{ij}^y - \gamma_{ij}^{y1}\right) a_{ij}^{y2} + \gamma_{ij}^{y2} a_{ij}^{y2} + \left(1 - \lambda_{ij}^y - \gamma_{ij}^{y2}\right) a_{ij}^{y3} \tag{9}$$

$$\bar{y}_{ij} = \gamma_{ij}^{y1} b_{ij}^{y1} + \left(\lambda_{ij}^y - \gamma_{ij}^{y1}\right) b_{ij}^{y2} + \gamma_{ij}^{y2} b_{ij}^{y2} + \left(1 - \lambda_{ij}^y - \gamma_{ij}^{y2}\right) b_{ij}^{y3} \tag{10}$$

$$0 \le \gamma_{ij}^{x1} \le \lambda_{ij}^x \tag{11}$$

$$0 \le \gamma_{ij}^{x2} \le 1 - \lambda_{ij}^x \tag{12}$$

$$0 \le \gamma_{ij}^{y1} \le \lambda_{ij}^y \tag{13}$$

$$0 \le \gamma_{ij}^{y2} \le 1 - \lambda_{ij}^y \tag{14}$$

$$\bar{x}_{ij} \ge \left(\frac{1}{2}\left(l_i^x + l_j^x\right) + d_{ij}^x\right) m_{ij}^x \tag{15}$$

$$\bar{y}_{ij} \ge \left(\frac{1}{2}\left(l_i^y + l_j^y\right) + d_{ij}^y\right) m_{ij}^y \tag{16}$$

$$m_{ij}^x + m_{ij}^y = 1. \tag{17}$$

Constraints 7–17 should be written for $i = 1 \cdots n-1,\ j = i+1 \cdots n$. A derivation of constraints 7–14 is provided in the Appendix.

$$\frac{1}{2} l_i^x \le x_i \le h - \frac{1}{2} l_i^x \qquad \text{for} \quad i = 1 \cdots n \tag{18}$$

$$\frac{1}{2} l_i^y \le y_i \le v - \frac{1}{2} l_i^y \qquad \text{for} \quad i = 1 \cdots n \tag{19}$$

$$x_i, y_i, \bar{x}_{ij}, \bar{y}_{ij} \ge 0 \qquad \text{for} \quad i = 1 \cdots n, \qquad j = i+1 \cdots n \tag{20}$$

$$\lambda_{ij}^x, \lambda_{ij}^y, m_{ij}^x, m_{ij}^y \in \{0, 1\}. \tag{21}$$

The formulation **FD1** can also be written for three-dimensional layout problems, by adding constraints similar to Eqs. 7, 8, 11, 12, and 13. Equation 17 should include a new binary variable m_{ij}^z. The two-dimensional formulation consists of $3/2n(n-1)$ binary variables and $5n^2 - n$ constraints, where n is the number of units. This is the case if Eq. 17 is used to eliminate the binary variables m_{ij}^y. On the other hand, **FL** has $n(n-1)$ binary variables and approximately 50% fewer

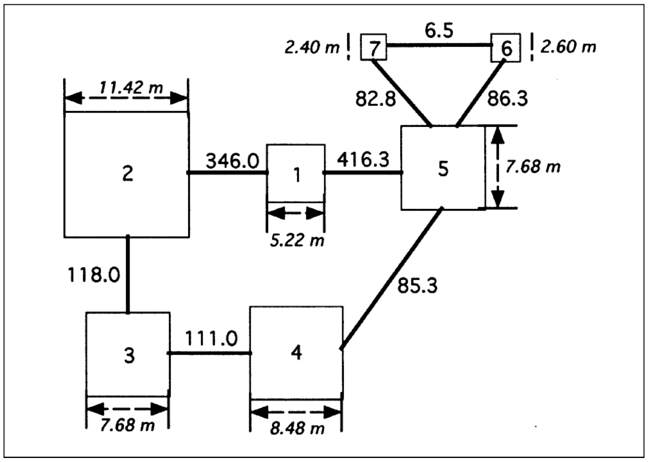


Figure 4. Equipment dimensions and connection costs (in rmu/m) for Example 1.

constraints. Thus in these formulations there is a trade-off between the size of the problem and the tightness of the linear relaxation—a trade-off that can only be explored empirically.

Layout Problem Examples

To test the performance of **FD1** we will attempt to solve different problems from the literature. The formulations are modeled using GAMS (Brooke et al., 1992), and the resulting mixed-integer linear programs are solved using CPLEX 4.0 (CPLEX, 1989–1995).

Solutions using formulation FD1

Example 1. The first example we solve is the ethylene oxide plant example from Papageorgiou and Rotstein's article (1998). The equipment will be laid out on a 30-by-30-m floorplan and the square dimensions and connection costs are given in Figure 4.

Table 1. Interaction Costs for the Pipeless Plant Layout Problem

Connection	Cost
(Charger 1, Reactor B)	24
(Charger 1, Mixer B)	6
(Charger 1, Cleaner 1)	15
(Charger 2, Reactor A)	25
(Charger 2, Mixer A)	6
(Charger 2, Cleaner 1)	15
(Reactor A, Mixer A)	25
(Reactor B, Mixer A)	20
(Reactor B, Mixer B)	24
(Mixer A, Cleaner 1)	15
(Mixer B, Cleaner 2)	15

After 626 branch-and-bound algorithm nodes and 5 s, **FD1** obtained a solution with an objective value of 9948.03 rmu with a 8.7% margin of optimality. After 72 s and 12,253 nodes, the gap between the objective value and lower bound became less than 5%. The resulting layout can be seen in Figure 5. The result found using **FL** is 9978.5 rmu after 35,100 nodes and 54.9 s, which is reported to be within a 5% margin of optimality (Papageorgiou and Rotstein, 1998). This shows that there can be an advantage in using a formulation that has relatively few variables and constraints because of the increased speed of solving the LPs at the nodes.

Example 2. The second problem that we solve is the pipeless-plant layout problem (Gonzales and Realff, 1998). A total of 8 units will be located by minimizing the total interaction cost between each unit, where interaction cost is measured by the number of journeys a vessel has to make between any given pair of stations. Every unit is 4 m by 4 m, and every pair of units is at least 2 m apart. The interaction costs are given in Table 1. The resulting layout, which is fitted in a 25-by-25-m area, can be seen in Figure 6.

A solution of 1212 rmu is achieved in 30 s at the 3294th node with a 6.3% gap using the **FD1** formulation. This problem was also solved using **FL**. With default settings of CPLEX (CPLEX, 1989–1995) a lower bound of 1030 rmu is achieved

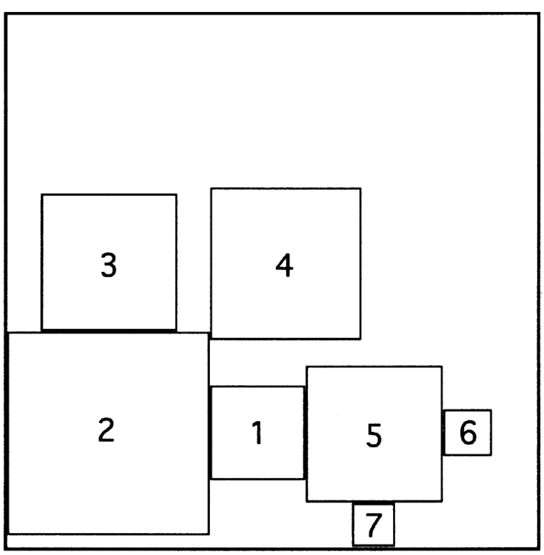


Figure 5. Optimal layout for Example 1.

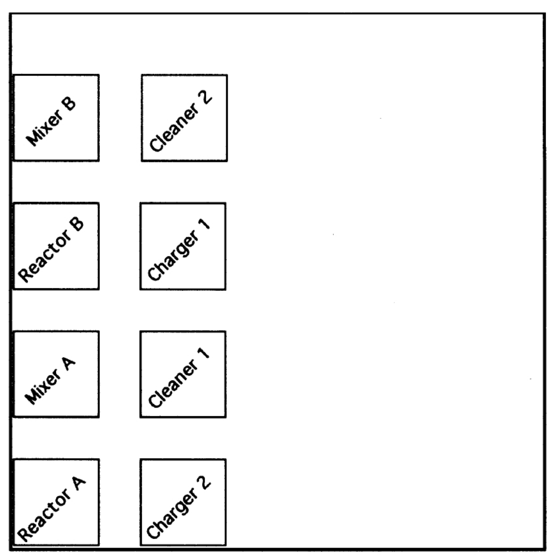


Figure 6. Optimal layout for Example 2.

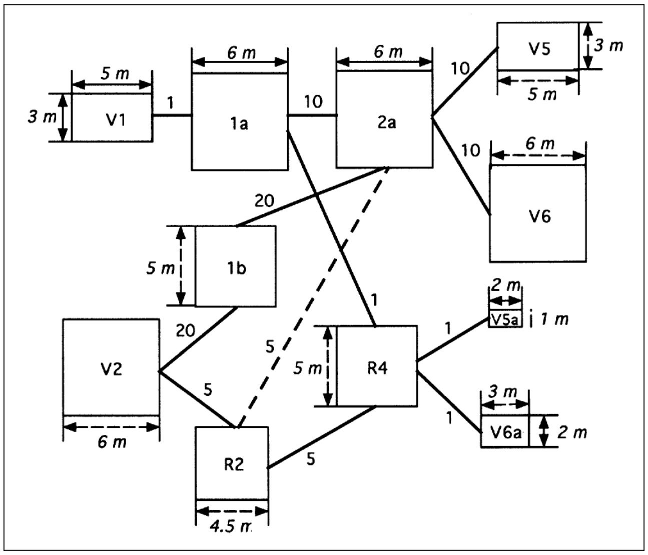


Figure 7. Equipment dimensions and connection costs (in rmu/m) for Example 3.

after 250 s, with a best-integer feasible solution of 1482 rmu (44% gap). The best-integer solution is improved to 1212 rmu by changing the node-selection option in CPLEX to best-estimate search (in the branch-and-bound algorithm, the selection of the next node is done according to the estimates of the objective function value that would result if all integer infeasibilities were to be removed), whereas the lower bound stayed at 760 after 250 s (60% gap). As can be seen **FD1** gives better lower-bound refinements and solutions for these two problems, which is to be expected due to its tighter approximation of the integer feasible hull. It is not possible to conclude that **FD1** will always yield better results from this limited empirical study or any other such study; rather, we should seek to combine the strengths of the two formulations. **FD1** usually finds better solutions faster and has better initial lower bounds; however, lower-bound improvement is rather slow. On the other hand, **FL** has fewer variables and constraints, which reduces the computation required for a given node. This can lead to better performance in overall algorithm terms because of the ability to examine more nodes in the same time as **FD1**.

Solutions using a hybrid formulation

Example 3. The data for the third problem are given in Figure 7. This is a modified version of the batch plant layout problem from Papageorgiou and Rotstein (1998). The only change in the problem definition is the addition of the interaction between units R2 and 2a, with a 5 rmu/m connection cost.

Both **FD1** and **FL** formulations are employed to describe this problem. **FD1** calculates 571 rmu as the upper bound and 477.75 rmu as the lower bound after 500 s, and the corresponding gap is 20%. **FL** finds 672 rmu in 500 s as the upper bound and 477.75 rmu as the lower bound, which corresponds to a 41% gap compared to a less than 5% gap in 6.3 s for the original problem (Papageorgiou and Rotstein, 1998).

In principle, the two formulations can be combined, since each one represents the same relationships in slightly different ways. This joint formulation might inherit some of the strengths of both **FD1** and **FL**, and hence be effective across a wider range of problems. A hybrid formulation is constructed by combining **FD1** with **FL**. The resulting model can be summarized as follows

FH
$$\min \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} C_{ij}\left(\bar{x}_{ij} + \bar{y}_{ij}\right), \tag{6}$$

subject to constraints 7–16, which are written for only connected units, constraints 18–21, and

$$x_i - x_j + M\left(1 + E1_{ij} - m_{ij}\right) \geq \frac{1}{2}\left(l_i^x + l_j^x\right) + d_{ij}^x \tag{22}$$

$$x_i - x_j + M\left(2 - E1_{ij} - m_{ij}\right) \geq \frac{1}{2}\left(l_i^x + l_j^x\right) + d_{ij}^x \tag{23}$$

$$y_i - y_j + M\left(E1_{ij} + m_{ij}\right) \geq \frac{1}{2}\left(l_i^y + l_j^y\right) + d_{ij}^y \tag{24}$$

$$y_i - y_j + M\left(1 - E1_{ij} + m_{ij}\right) \geq \frac{1}{2}\left(l_i^y + l_j^y\right) + d_{ij}^y, \tag{25}$$

where constraints 22–25 should be written for unit pairs that are not connected.

Constraints 22–25 are adopted from nonoverlapping constraints of **FL** where M is a sufficiently large number (a reasonable choice is $M = \max\{h, v\}$) and $E1_{ij}$ describe whether unit i is to the right of (above) or to the left of (below) unit j. Using this formulation, a solution is found in 22 s after 2238 nodes as 505.5 rmu, which is within 5% of the lower bound. The resulting layout can be seen in Figure 8, where the floor plan is a 35-by-30-m rectangle. If the minimum allowable distance between all units is changed to 2 m, the result 658.75

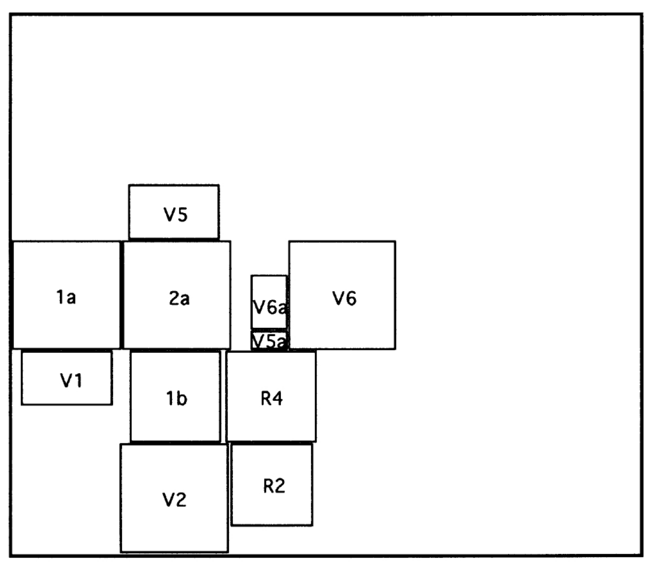


Figure 8. Optimal layout for Example 3.

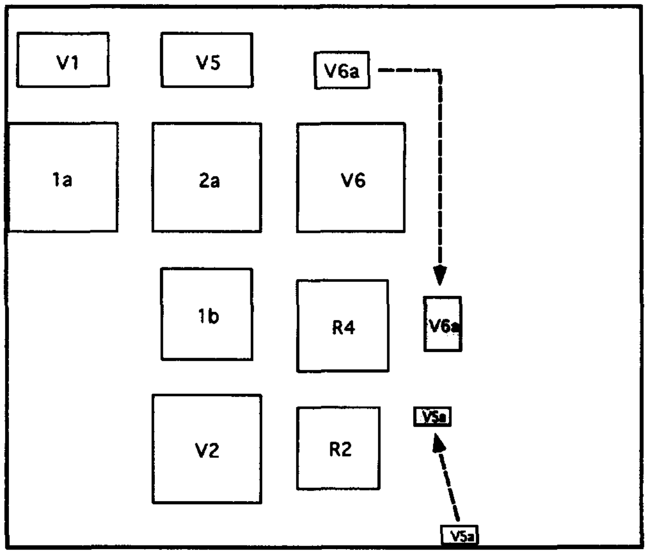


Figure 9. Optimal layout for Example 3 (with minimum allowable distance of 2 m between all units).

rmu is found after 13,397 nodes and approximately 100 s, and since the lower bound increased slowly, the solution had a 10% gap after 350 s. Figure 9 shows that obvious improvements can be made to lower the integrality gap. Improvement in the solution is obtained by fixing all units but V5a and V6a to their locations obtained from initial solution. This result, with an objective function value of 642 rmu, is shown by shaded units V5a and V6a.

Discussion

It is clear that even simple changes in the problem data can make a given combination of formulation and the branch-and-bound procedure fail to close the gap between upper and lower bounds. This is the practical outcome of the NP-hard results for combinatorial optimization problems (Garey and Johnson, 1979), highlighted in Pekny and Reklaitis (1998). Although the hybrid formulation seems to give good results for the described chemical engineering applications, it fails to solve problems with dense interaction matrices (Nugent et al., 1968). These kinds of flows, however, are common for batch and/or pipeless plants. Moreover, it is well known that there is a relation between flow matrix and complexity of facility layout problems (Herroelen and van Gils, 1985). Thus to solve layout problems reliably in a variety of chemical-engineering applications there are two basic approaches. One is to develop high-quality heuristics with known performance bounds (Bertsimas and Teo, 1998) both in terms of quality of solution and running times polynomial in the size of the problem. The other is to develop customized formulations and enumeration schemes that reflect the particular application constraints and data ranges (Kirschner and Realff, 1999).

An important element of any strategy is to make the best use of information that might provide constraints on the overall problem solution. This information can help break symmetries in the layout, which often results in many equivalent solutions being enumerated by a rigorous search algorithm, or provide leverage for the synthesis of effective heuristics. One source of information is the geographic location of the proposed layout. In the next section GISs are introduced as a way to capture, manipulate, and disseminate this type of information.

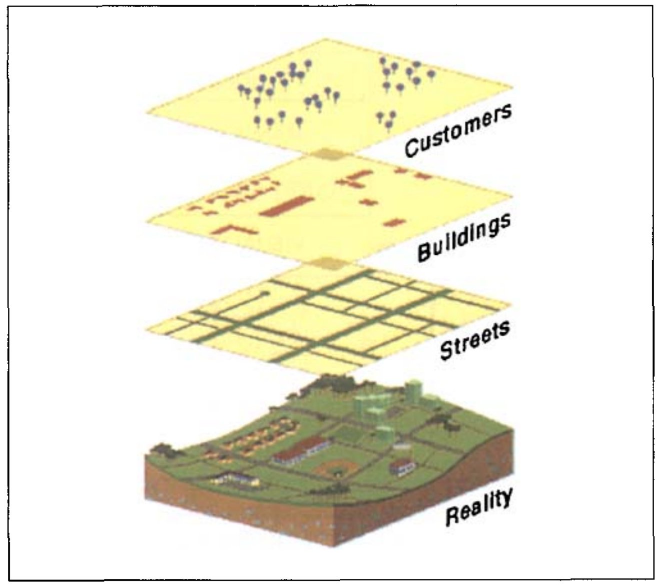


Figure 10. Collection of thematic layers to present the reality. (Graphic image supplied courtesy of Environmental Systems Research Institute, Inc.)

Geographic Information Systems and Their Application

GIS is a computer-based mapping and analysis tool. Its ability to integrate common database operations with a unique capacity for the visualization and geographic analysis of maps makes GIS valuable as a decision support system. In a GIS the real world is represented by a collection of thematic layers that can be used separately or together to elicit knowledge about specific locations, as depicted in Figure 10. Explicit geographic references, such as x- and y-coordinates or latitude and longitude, can be created from implicit references like addresses or facility names.

The location-specific knowledge is stored as attribute data in a database, which is the most important component of GIS. This database is assembled using data input and manipulation tasks. Many types of geographic data can be obtained from data suppliers and loaded directly into a GIS. It is likely that the data obtained from different sources are available at different degrees of detail or accuracy. Before this information can be integrated, it must be manipulated to the same scale. Once a functioning GIS containing geographic information is obtained, query tasks can be performed. Questions such as Where is land zoned for industrial use? and How far is it between two places? can be answered easily. Likewise the analysis task helps the user find answers for the following types of questions:

- Where are all the sites suitable for building new production facilities?
- If I build a poultry facility here, where are the possible discharge points for wastewater?

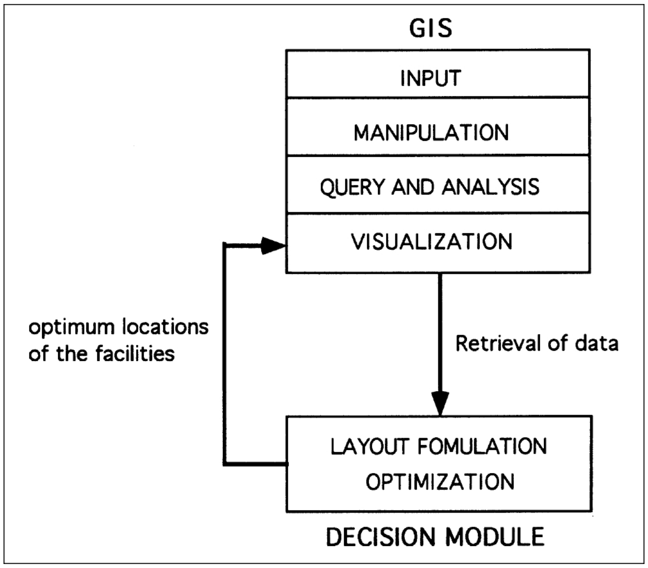


Figure 11. Methodology for the integration of GIS with layout decisions.

• How many facilities lie within 100 m of this water main? The results of any query and analysis are usually represented as a map or graph. After visualizing the current situation, further manipulation and analysis of the input can be undertaken.

The focus of this section is the addition of a decision module to a GIS to add a synthetic component to an otherwise purely analytical tool. We have chosen mathematical programming as a decision tool, and specifically a MILP representation. This is a compromise between the compatibility of MILP with the representation of the real objects within a GIS, and the need for reasonable computational performance. Based on analytic geometry and GIS, the model formulation can be improved in several ways. First, the spatial data can help define the problem by giving the system boundary and also information about the surroundings and existing units. Second, GIS analysis tools can be used to generate constraints for the problem. Before going into details of these interactions between GIS and decision module (Figure 11), we can describe the general idea in the context of the plant layout problem.

By applying data input and manipulation tasks of GIS, information relevant for the chemical facility layout problem can be prepared for further analysis. This information includes existing units or facilities, soil, elevation, climate information, maps containing roads, rivers, and lakes. Moreover, emergency routes, security issues, and geographic factors like seismic disturbance, ground contours, and water-table level can also be added. Utilizing this information, GIS will help to visualize the real situation, detect constraints on a map, and give attribute data about the individual units and their environment. The existing plant layout can be given as an AutoCAD file (Yarwood, 1994), containing location and dimension information. Another possible way to convey the data is using images of the facility, which can be incorporated with the attribute data available for the objects on the image. This is possible only for facilities in which the locations of units can be determined from satellite or aerial photography such as units not obscured by roofing. The data required by the decision module will be passed to an MILP solver via a modeling language, such as GAMS (Brooke et al., 1992), and the mathematical program described in previous sections will be optimized. The resulting location information can be visualized using x- and y-coordinates as implicit references. The layout containing new equipment or facilities that are to be sited on a given area can be used for further analysis and evaluation by using the query and analysis tasks of GIS or by other external analysis tools. For example, a Voronoi diagram (Okabe and Suzuki, 1997) can be created to plan emergency routes.

In a GIS, which uses a list of edge endpoints instead of pixels to model the geographic features, spatial objects are described by points, lines, and polygons (Figure 12). Points, besides defining lines and polygons, give the x-, y-coordinate information of spatial objects like process units and warehouses when their finite size is unimportant. Lines or their collections or arcs can specify roads, rivers, and railways. Finally, polygons are helpful in representing areas such as process units, floor plans, plant boundaries, census tracts, or flood patterns. More generally, there are scalar and vector quantities whose magnitude and direction depend on their location. A subset of these features, namely constant vector fields and polygonal scalar fields (Figure 13), can be used to represent additional information about the area of interest like elevation and prevailing wind direction and speed.

Geometrical analysis using points, lines, and polygons results in single or multiple (non)convex polygons that transfer

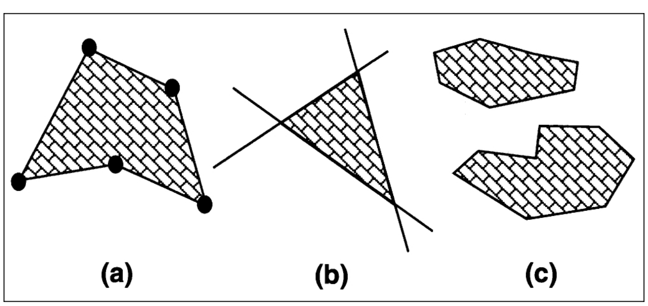


Figure 12. Different types of (in)feasible-region generation using basic GIS coverages: (a) points; (b) lines; (c) polygons.

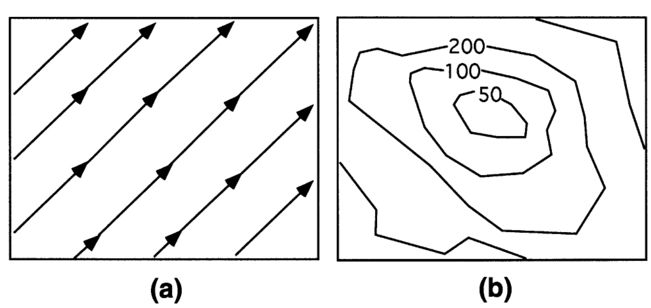
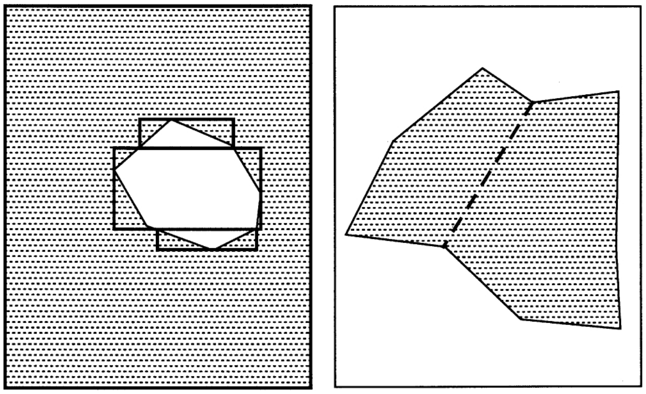


Figure 13. Two types of fields: (a) constant vector fields; (b) polygonal scalar fields.



(a) (b)

Figure 14. Infeasible (a) and feasible (b) regions.

to the mathematical programming formulation as feasible and infeasible regions. For instance, if we are considering several units described by points and want to locate new units within the hull of these existing units, the feasible region, in general, will be a nonconvex polygon. Buffer zones around roads, closeness to emergency action locations, or soil type criteria for the site selection can be described by polygonal feasible regions. There are two main problems with these regions. The first is infeasible holes within the feasible region. The infeasible holes, which contain non-Manhattan edges, that is, not rectilinear polygons, can be approximated by rectangular polygons and then partitioned into nonoverlapping rectangles. Although this dissection can be done visually, an algorithm for minimum partitioning can also be used (Ohtsuki, 1982). The resulting rectangles can be included in the layout problem as pseudoprocess units. Since the formulation doesn't allow overlapping of units, the infeasible hole will be eliminated from the feasible region (Figure 14a). Although we are excluding some feasible portions by this approximation, the burden of dealing with several disjunctions within the MILP model is eliminated. The second is compact feasible regions with nonconvex boundaries. These can be partitioned into convex ones visually using GIS mapping capabilities (Figure 14b), or polygon partitioning algorithms: the Hertel-Mehlhorn algorithm can be used to find nonoverlapping convex polygons (O'Rourke, 1998). Disjunctions of convex or convexified compact feasible regions can be described by the vertices of the polygons and formulated in an MILP as disjunctive constraints for some or all units. For **P** disjoint feasible polygon regions defined by K vertices $(x_1^p, y_1^p), \dots, (x_k^p, y_k^p), \dots, (x_K^p, y_K^p)$, the equations describing polygons can be written as

$$l_k^p x_k^p + m_k^p y_k^p + n_k^p \leq 0 \; \forall p,$$

where

$$l_k^p = y_k - y_{k+1}, \qquad m_k^p = x_{k+1} - x_k,$$
$$n_k^p = x_k y_{k+1} - x_{k+1} y_k.$$

The disjunctions are then

$$\bigcup_p \{L^p x + M^p y + N^p \leq 0\},$$

which can be written as follows (Lowe, 1984)

$$x = \sum_{p=1}^{P} \tilde{x}_p$$

$$y = \sum_{p=1}^{P} \tilde{y}_p$$

$$L^p \tilde{x}_p + M^p \tilde{y}_p + N^p \tilde{z}_p \leq 0 \; \forall p$$

$$\sum_{p=1}^{P} \tilde{z}_p = 1$$

$$\tilde{z}_p \in \{0, 1\}.$$

In summary, noncompact, nonconvex feasible regions induced by 2-D geometrical objects such as points, lines, and polygons can be handled by the following procedure. First, the infeasible holes are eliminated by covering them with rectangles and considering these rectangles to be existing units. Second, the feasible regions that are now compact, are examined for the nonconvexity of their boundaries. Any nonconvex region is partitioned into convex subregions. Third, the set of convex feasible subregions is represented using disjunctive constraints.

The constraints deduced from fields can be used straightforwardly before or after the optimization in a pre- or postprocessing phase. However, to utilize them as part of the MILP formulation can be cumbersome. For example, for polygonal scalar fields, the contours should be convexified and for every unit affected by these contours the location relative to these convexified regions will be calculated during the optimization procedure through an extensive set of additional constraints. A simpler case is that of constant vector fields. These can be formalized as follows: slope of the field lines $= s$; intercept with the y-coordinate axis $= \beta$; for an arbitrary point describing the center of a unit,

$$y_1 = s x_1 + \beta$$

describes the field line passing through that point (x_1, y_1):

$$\beta = s x_1 - y_1$$

If we define a range for s as Δs, then $\Delta \beta = \Delta s \, x_1$, and will represent a cone that will be infeasible for a specific unit or several units. In other words, this unit's center coordinates, (x, y), should satisfy the following disjunctive constraints:

$$y \leq (s + \Delta s)x + s x_1 - y_1 + \Delta \beta$$
$$x \leq x_1,$$
or
$$y \geq (s - \Delta s)x + s x_1 - y_1 - \Delta \beta$$
$$x \leq x_1, \quad \text{or} \quad x \geq x_1$$

These constraints are applied to a 4-unit symmetric system for two units (units 1 and 3), for which the interaction matrix

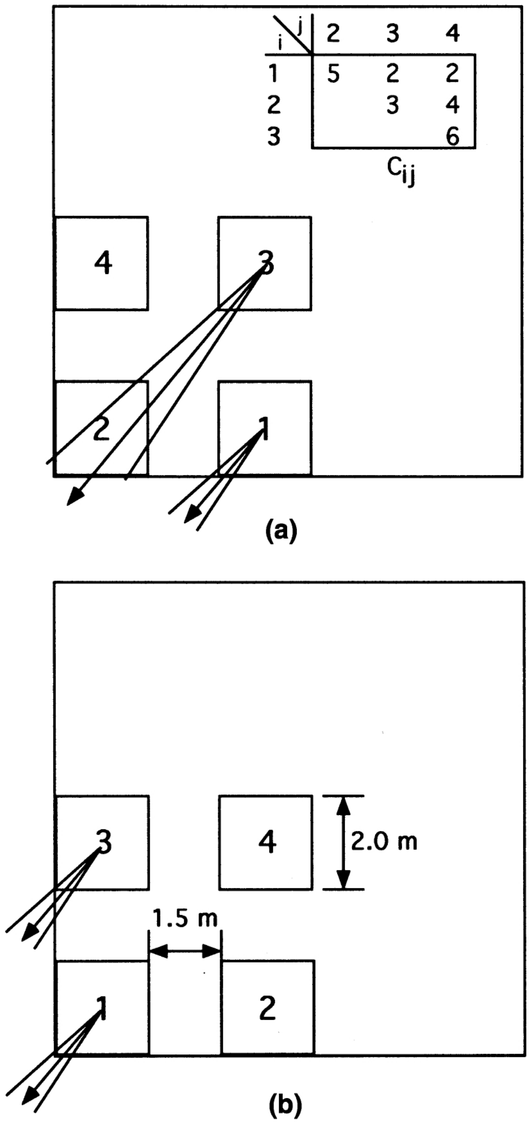


Figure 15. Solution of the 4-unit system without (a) and with (b) field information.

and the solution without the field information are given in Figure 15a. This inclusion of field information increases the number of constraints from 89 to 127 and the cumulative number of iterations from 6880 to 7138, but reduces the number of branch-and-bound algorithm nodes from 1648 to 1517.

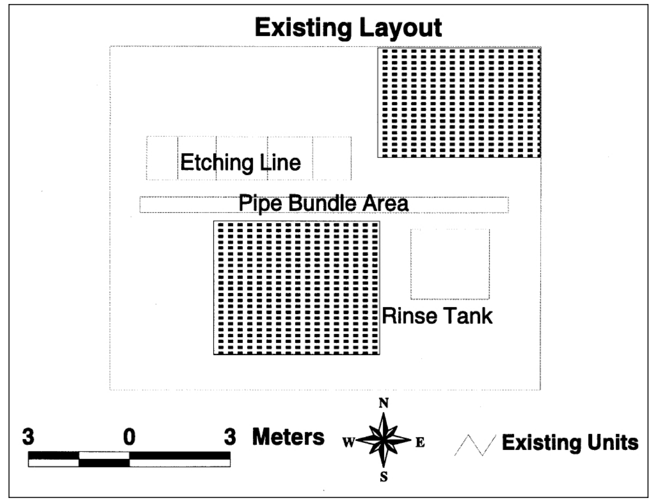


Figure 16. Current layout and feasible regions in which to locate the center of LIX 63 tank.

We will illustrate these concepts for retrieving data from GIS with the methodology through two case studies in the following section.

Case Studies

Case Study 1

In a paper by El-Halwagi and Manousiouthakis (1989), a mass exchange network (MEN) was designed for an etching plant with an etching line and a rinse bath. The objective was to recover copper from the process water, using two different extractants, LIX63 and P1 (two hydroxyoximes). The existing equipment locations are depicted in a simple layout that is generated as an AutoCAD file (Yarwood, 1994) and converted into an Arcview shapefile (*Arcview GIS*, 1998). The resulting map can be seen in Figure 16. The tabular data, such as location, size, and unit code, are deduced from the existing plant description and MEN synthesis results and entered as the main attribute table for the current layout layer (Table 2). The pumping cost is assumed to be the only connection cost that is given as separate data (Table 3) and linked to the main attribute table. For a given unit, the connection cost to other units and the minimum allowable distance data (Table 4) can be easily acquired.

The information needed for the optimization in the decision module is captured and used as input data to the layout problem formulation. The two extractant tanks and three

Table 2. Main Attribute Table for Case Study 1

Code	Unit	Mid_Loc_x	Mid_Loc_y	Length_x	Length_y
EL1	Etching Line	4.14	6.95	6.15	1.28
PL1	Pipe Bundle Area	6.39	5.56	11.00	0.45
T101	Rinse Tank	10.18	3.77	2.35	2.10
FL	Floor Boundary	6.44	5.13	12.86	10.24
C103	Rinsewater Column 2	0.00	0.00	1.00	1.00
C102	Rinsewater Column 1	0.00	0.00	1.00	1.00
C101	Etchant Column	0.00	0.00	1.00	1.00
T102	LIX63 Tank	0.00	0.00	2.00	2.00
T103	P1 Tank	0.00	0.00	1.50	1.50

Table 3. Connection Cost Data for Case Study

Connection	Cost ($/m)
T101, C102	16
T102, C101	54
T102, C102	54
T103, C103	4
C101, EL1	40
C101, T102	54
C102, T101	16
C102, T102	54
C102, C103	16
C103, T103	4
C103, C102	16

columns for extraction are located, as shown in Figure 17, with a minimum connection cost of $752.32. (This result is found after 576 s and 192,077 branch-and-bound algorithm using formulation **FH**. A corresponding gap of the solution is 0.5%.) If the two feasible regions depicted as two shaded areas in Figure 16 are considered for possible locations of the center point of LIX 63 Tank, then the same solution can be found after 255 s and 80,850 nodes. This new layout is stored in GIS for future use. As an example the unit coordinates are extracted from GIS and used to determine the route from which the distance of the nearest unit is the longest by using area Voronoi diagrams (Okabe and Suzuki, 1997). This analysis can be useful when planning escape routes.

Case Study 2

This example shows the ability of GIS to represent and coordinate information about the physical surroundings of a location. The plant layout problem involves three tasks, namely site selection, site layout, and plot layout. Until now we have dealt with plot layout issues, although GIS can be of value to the decision maker in site selection and site layout. This case study will illustrate the interaction between site layout and plot layout decisions.

Suppose that the ethylene oxide plant in Example 1 is to be laid out in the yellow area in Figure 18. An airborne image is used to identify existing plants and equipment and this is entered into the problem database. Land-use information and elevation data are also prepared for the analysis. As a result of the analysis, elevation data, location of the oxygen

Table 4. Minimum Distance between Units for Case Study 1

Unit Pairs	Distances (m)
T101, C103	2
T102, T103	1
T102, C101	1
T102, C102	1
T102, C103	1
T103, C101	1
T103, C102	1
T103, C103	1
C101, C102	3
C101, C103	3
C102, C103	3

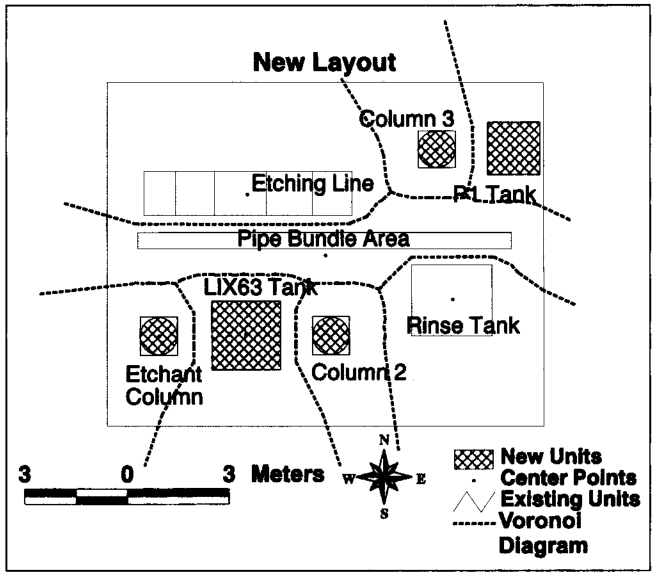


Figure 17. New layout and arcs that maximize the distance to any unit.

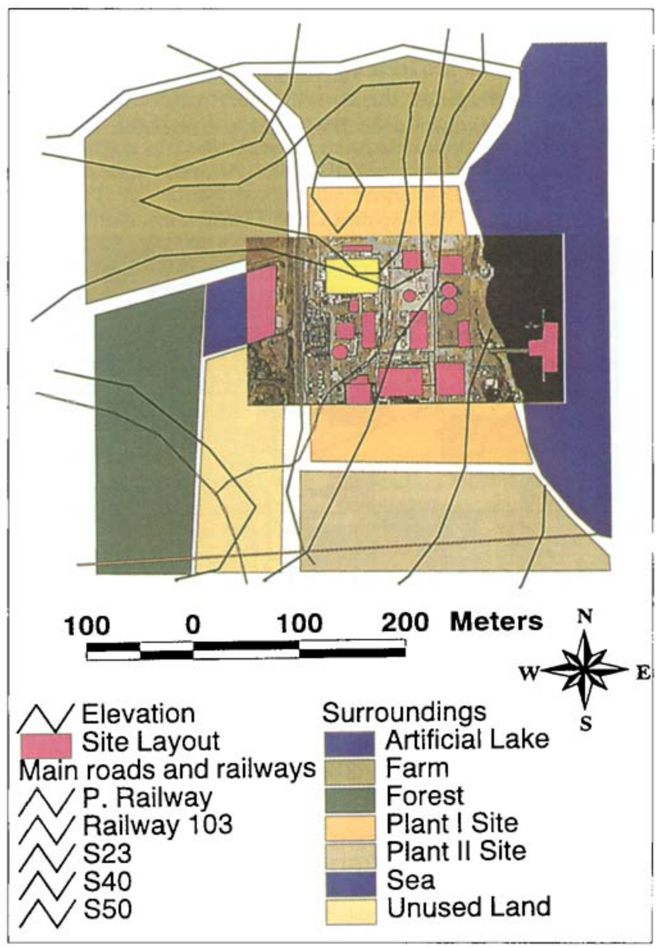


Figure 18. Aerial photograph and thematic layers describing plant and its surroundings.

plant, and land-use information justify the restriction of the reactor being at the northeast corner of the floor plan. This leads to the following constraints added to the layout formulation:

$$y_R \geq -1.16x_R + 54.7 \quad (26)$$

$$y_R \geq 21, \quad (27)$$

where the subscript R indicates the reactor.

On the other hand, we require unit 3 (ethylene oxide absorber-EOA) to be close to the tank yards:

$$y_{\mathrm{EOA}} \leq 16. \quad (28)$$

The information obtained from GIS results in the layout in Figure 19. This layout, which also uses the minimum allowable distances obtained from the optimal solution in Penteado and Ciric (1996), namely 24, 22, and 23 meters between the reactor and EO absorber, reactor and CO_2 absorber, and both absorbers, respectively. The optimum cost is found to be 24,072.127 rmu, which corresponds to an integer solution, proved to be optimal after 18 s and 5678 nodes. The addition of location-specific information reduced the computation time from 54.9 s to 18 s and the number of nodes evaluated from 35,100 to 5678.

Once the units are located, a simple consequence analysis can be performed and the contours for overpressure caused by a possible explosion in the reactor calculated (Skelton,

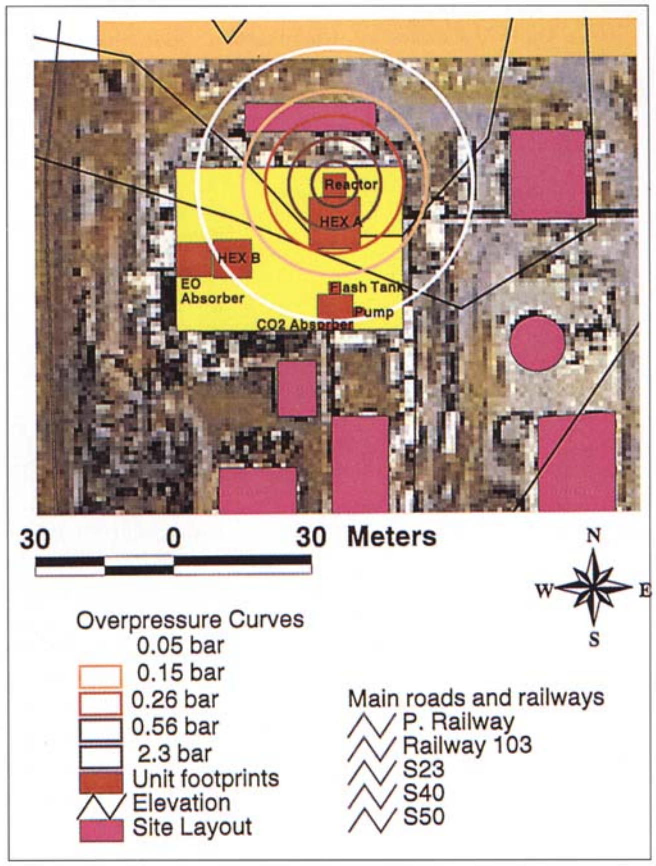


Figure 20. Contour showing the overpressure generated by an explosion in the reactor.

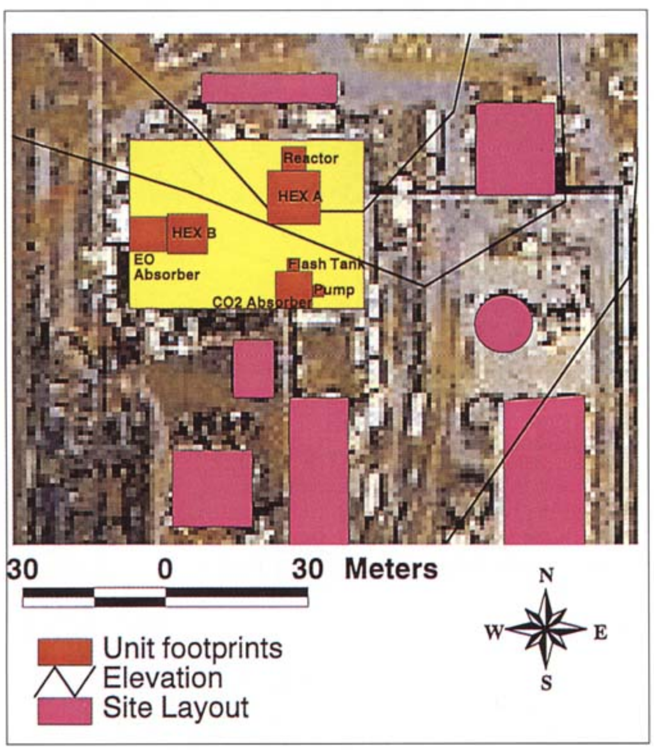


Figure 19. Layer showing the resulting locations of equipment footprints.

1997). As can be seen in Figure 20, most of the units are located apart from the reactor, which is the most explosion-prone piece of equipment in the plant.

Conclusions

This article emphasizes the integration of information systems with decision-making procedures. The use of geographic information systems (GIS) as a decision support system for plant layout problem has been demonstrated. Besides supplying the relevant data required to lay out the facility isolated from its surroundings, GIS can help the decision-making procedure to draw qualitative and quantitative conclusions from the real environment where the facility is located. This new knowledge can be used to add constraints to the problem by restricting the locations of some units before applying any optimization.

An MILP-type continuous-plane formulation of the layout problem has been developed and can be useful and efficient in problems where the number of units is relatively small. An improvement of the current formulations was developed by hybridizing two different schemes for representing disjunctive information and absolute values. This approach was able to solve a wider range of problems, but depending on the problem data, there are subclasses that are not solved efficiently. Two case studies showed the advantages of utilizing a geographic information system for the plant layout problem in combination with the hybrid formulation.

The integration of GIS and other decision-making methodologies may improve the overall performance of the system. For example, the use of metaheuristics such as genetic algorithms, tabu search, and simulated annealing (Castell et al., 1998; Welgama and Gibson, 1995; Barr et al., 1997) may provide good solutions quickly and optimal ones in the limit. GIS–optimization integration can also facilitate integrated design, layout, and off-site decision making. The use of 3-D versions of the mathematical formulations coupled with 3-D GIS can enhance the methodology to cover the layout problems involving multilevel buildings.

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Notation

Parameters

C_{ij} = cost of connecting unit i to unit j, or flow between units i and j
d_{ij}^x = minimum allowable distance between units i and j in the x-direction
d_{ij}^y = minimum allowable distance between units i and j in the y-direction
h = length of the floor area
v = width of the floor area

Variables

l_i^x = length of unit i
l_i^y = width of unit i
x_i = x-coordinate of the center of unit i
y_i = y-coordinate of the center of unit i

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Appendix

The representation for the graph of a piecewise-linear function depicted in Figure 3 can be derived as follows (Jeroslow, 1989).

Let a point in 2-D space with coordinates x_{ij}, $\bar{x}_{ij}$ be represented by $(x_{ij}, \bar{x}_{ij})$:

$$P_1 \equiv \Big\{ (x_{ij}, \bar{x}_{ij}) \,|\, \text{for some } \gamma_{ij}^{x}, 0 \le \gamma_{ij}^{x} \le 1, \quad (x_{ij}, \bar{x}_{ij}) = \gamma_{ij}^{x}\big(a_{ij}^{x1}, b_{ij}^{x1}\big) + \big(1-\gamma_{ij}^{x}\big)\big(a_{ij}^{x2}, b_{ij}^{x2}\big) \Big\}$$

$$P_2 \equiv \Big\{ (x_{ij}, \bar{x}_{ij}) \,|\, \text{for some } \gamma_{ij}^{x}, 0 \le \gamma_{ij}^{x} \le 1, \quad (x_{ij}, \bar{x}_{ij}) = \gamma_{ij}^{x}\big(a_{ij}^{x2}, b_{ij}^{x2}\big) + \big(1-\gamma_{ij}^{x}\big)\big(a_{ij}^{x3}, b_{ij}^{x3}\big) \Big\}.$$

Thus $P_1 \cup P_2$ can be represented by

$$\big(x_{ij}^{(1)}, \bar{x}_{ij}^{(1)}\big) = \gamma_{ij}^{x1}\big(a_{ij}^{x1}, b_{ij}^{x1}\big) + \big(\lambda_{ij}^{x(1)} - \gamma_{ij}^{x1}\big)\big(a_{ij}^{x2}, b_{ij}^{x2}\big), \quad 0 \le \gamma_{ij}^{x1} \le \lambda_{ij}^{x(1)}$$

$$\big(x_{ij}^{(2)}, \bar{x}_{ij}^{(2)}\big) = \gamma_{ij}^{x2}\big(a_{ij}^{x1}, b_{ij}^{x1}\big) + \big(\lambda_{ij}^{x(2)} - \gamma_{ij}^{x2}\big)\big(a_{ij}^{x2}, b_{ij}^{x2}\big), \quad 0 \le \gamma_{ij}^{x2} \le \lambda_{ij}^{x(2)}$$

$$\big(x_{ij}, \bar{x}_{ij}\big) = \big(x_{ij}^{(1)}, \bar{x}_{ij}^{(1)}\big) + \big(x_{ij}^{(2)}, \bar{x}_{ij}^{(2)}\big)$$

$$\lambda_{ij}^{x(1)} + \lambda_{ij}^{x(2)} = 1 \quad \text{and} \quad \lambda_{ij}^{x(1)}, \quad \lambda_{ij}^{x(2)} \in \{0,1\},$$

which simplifies to constraints 7, 8, 11, 12 if $\lambda_{ij}^{x} = \lambda_{ij}^{x(1)}$.

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